

## BENDABLE KB TYPE FOCUSING MIRRORS DESIGNED FOR TPS IR BEAMLINE

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### Abstract

A new IR beamline has been scheduled at TPS beam-line construction Phase III. The new beamline optical design is following the structure of the existed TLS IR beamline. However, the focusing mirrors has to be re-deign according to different situation. These KB type mirrors (HFM and VFM) are same thickness flat stainless plates assembled with bending arms and bended with single motor each to fit quintic polynomial surface profiles for focusing and also modifying arc source effect of bending section. For a same thickness plate in addition with the bending arms effect to form a desired polynomial surface profile, it demands specific width distribution. With the drawing method and FEM iteration simulation, the optimized surface polynomial equation and width distribution design of the mirror plates were defined. The detailed design sequences will be described in this paper.

### INTRODUCTION

There is an IR microscopy beamline located at 14A branch in TLS NSRRC [1, 2]. However, the TLS is scheduled to be shut down after TPS phase III due to the budget consideration. A new IR beamline is therefore scheduled at phase III construction plane. With different conditions from TLS, The TPS IR beamline has to be re-design. The TPS IR beamline adopts the similar design as TLS by using K-B type focusing mirrors in the pre-focusing period. Two Stainless flat mirror plates are to be bended to the desired surface profile. In experience, a 5<sup>th</sup> order polynomial surface profile is enough and also for the manufacturing consideration [3].

With the frontend space consideration, HFM is located 2350mm from the light source point and 4150mm from the focus point. VFM is located 3900mm from the source point and 2600mm from the focus point respectively as in Fig. 1. The beam divergence angles of horizontal and vertical are 50mrad and 25mrad from the source point, respectively. The light-reflecting areas of HFM and VFM are about 166mm x 66mm (maximum) and 77mmx138mm respectively.

For an ideal point light source, by using an elliptical mirror, the light from one focus will be reflected and concentrated in another focus. This phenomenon is well adopted in VFM because it can be regard as from a point source without considering the electron beam size.

For HFM, the light source is an arc section from bending magnet, the light will not concentrate in another focus as in the Fig. 2 drawing and the profile should be modified.

In TLS HFM design, a fourth order RungeKutta numerical method was used to find out the coefficients of the

modified polynomial equation. This method is somewhat complicated to implement. A drawing method was adopted any try to easily find out the profile polynomial.

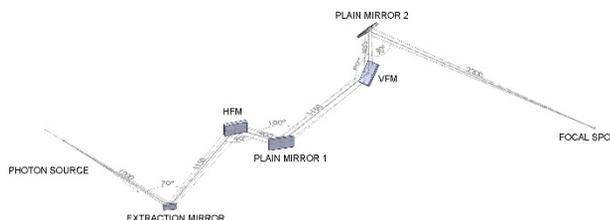


Figure 1: TPS IR beamline pre-focusing period mirror location scheme.

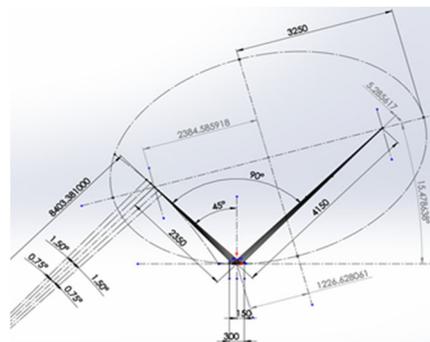


Figure 2: Drawing of light line from arc light source reflecting span on the focus point.

### IDEAL KB TYPE MIRROR SURFACE PROFILE EQUATIONS DERIVED WITH DRAWING METHOD

Let the required 5<sup>th</sup> order profile polynomials equation is (HFM & VFM both):

$$y(x) = c_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 \quad (1)$$

For HFM, at first, 5 ellipses were setup according to the arc divergence angle. The crossed sections were jointed to form a new profile and fitting to get a new polynomial as in Fig. 3.

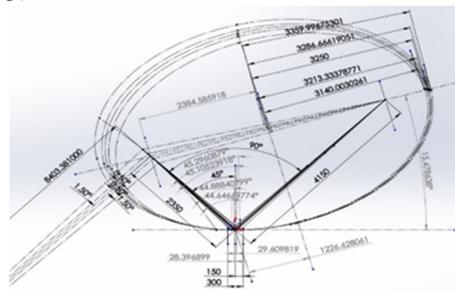


Figure 3: 5 ellipses setup according to the arc divergence angle.

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The coefficients of the fitting polynomial are listed in Table 1. With this polynomial, the focusing area as in the Fig. 2 can be replot and focusing size is reduced to 0.5mm. It's better but still not good enough.

Fortunately, from the polynomial equation, it can be observed with magnifying the focusing area on the drawing. Coefficient of the 2<sup>nd</sup> order term control the 4 lines concentration and coefficient of the 4<sup>th</sup> order term enlarge the outside 2 lines concentration rate. Coefficient of the 3<sup>rd</sup> order term control 4 lines inclination and coefficient of the 5<sup>th</sup> order term enlarge the outside 2 lines inclining rate.

With these conditions, an ideal HFM surface profile polynomial can be obtained. The optimized coefficients are listed in Table 1 and a replot of Fig. 2 shows the focusing area size is only 0.15um.

While the VFM surface profile polynomial can be found from an ideal elliptical section fitting. The coefficients are also listed in Table 1 and a drawing shows focusing area size is 0.52um.

Table 1: The Coefficients of Surface Polynomial

Coef.	5 ellipse section HFM	Optimized HFM	Ellipse VFM
c <sub>2</sub>	1.179797E-04	1.178178E-04	1.133183E-04
c <sub>3</sub>	-3.20187E-08	-3.4673E-08	5.136405E-09
c <sub>4</sub>	2.45467E-11	3.953E-11	3.090696E-12
c <sub>5</sub>	-4.2178E-14	-4.92E-14	3.998784E-16

### EQUATIONS DERIVED FOR BENDING FLAT MIRROR TO FIT IDEAL MIRROR SURFACE PROFILES

Although the ideal mirrors surface profile equations were obtained, the design requirement is to use a single force to bend a stainless flat mirror plate and to from the desired shape.

From the flexure equation:

$$\kappa(\text{curvature}) \approx \frac{d^2y}{dx^2} = \frac{M}{EI}$$

$$\Rightarrow M = EI(2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3) \quad (2)$$

Since the moment distributed of a same thickness plate from a single force is constant, a different width design is required to obtain the desired face shape. In principle, the flat plate with a 3<sup>rd</sup> order polynomial width distribution can get a surface profile polynomial of 5<sup>th</sup> order. But because the high order terms are not included in the calculation, to prevent the error accumulation, a 4<sup>th</sup> order polynomial width distribution can get a better approximation value.

Let the width distribution polynomial equation is:

$$b(x) = b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4 \quad (3)$$

The moment of inertia is:

$$I_t(x) = \frac{b_t(x)h_0^3}{12}$$

$$= \frac{I_0}{b_0}(b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4) \quad (4)$$

The bending moment is:

$$M = E \frac{I_0}{b_0} (b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4)(2c_2 + 6c_3x + 12c_4x^2 + 20c_5x^3)$$

$$= E \frac{I_0}{b_0} [2c_2b_0 + (6c_3b_0 + 2c_2b_1)x + (12c_4b_0 + 6c_3b_1 + 2c_2b_2)x^2 + (20c_5b_0 + 12c_4b_1 + 6c_3b_2 + 2c_2b_3)x^3 + (20c_5b_1 + 12c_4b_2 + 6c_3b_3 + 2c_2b_4)x^4 + \dots] \quad (5)$$

The bending moment is constant, let the coefficient of the variable terms to zero and get the coefficients of width polynomial as following:

$$b_1 = -3c_{32}b_0(c_{32} = c_3/c_2)$$

$$b_2 = -3c_{32}b_1 - 6c_{42}b_0(c_{42} = c_4/c_2)$$

$$b_3 = -3c_{32}b_2 - 6c_{42}b_1 - 10c_{52}b_0(c_{52} = c_5/c_2)$$

$$b_4 = -3c_{32}b_3 - 6c_{42}b_2 - 10c_{52}b_1 \quad (6)$$

However, the above calculation is the ideal situation of a single flat mirror plate bended with equal moment. The real design of the focusing mirror needs to be assembled with bending arms to apply the torque, the actual inertial moment is complex and difficult for theoretical analysis.

A flexible way is to make the moment of inertia of the mirror structure same as a single plate, the mechanism of the bending arm is another pseudo bending moment. With the same applied force, the pseudo moment distribution will be the same, so the desired surface curve profile can be approached by the plate width modification iteration without to know the real pseudo moment distribution.

The mirror width curve parameters calculated from the above initial calculations can be used to establish a basic model for finite element analysis by using Solidworks and COSMOS software.

The surface center curve from the FEM simulation is polynomial fitting as:

$$y_s(x) = s_2x^2 + s_3x^3 + s_4x^4 + s_5x^5 \quad (7)$$

$$\kappa_s(\text{curvature}) \approx \frac{d^2y_s}{dx^2} = \frac{M_s}{EI_t}$$

$$= 2s_2 + 6s_3x + 12s_4x^2 + 20s_5x^3 \quad (8)$$

$$M_s(x) = E \frac{I_0}{b_0} (b_0 + b_1x + b_2x^2 + b_3x^3 + b_4x^4)$$

$$(2s_2 + 6s_3x + 12s_4x^2 + 20s_5x^3) \quad (9)$$

A single force only affects the 2<sup>nd</sup> order term, the desired surface polynomial is:

$$y_r(x) = s_2x^2 + c_3x^3 + c_4x^4 + c_5x^5 \quad (10)$$

The correction of other terms of the surface polynomial requires modifying the coefficient of the width curve polynomial so that the inertial moment of the new width curve polynomial becomes:

$$I_n(x) = \frac{b_n(x)h_0^3}{12}$$

$$= \frac{I_0}{b_0}(b_0 + b_{1n}x + b_{2n}x^2 + b_{3n}x^3 + b_{4n}x^4) \quad (11)$$

$$\kappa_r(\text{curvature}) \approx \frac{d^2 y_r}{dx^2} = \frac{M_s}{EI_n} = 2s_2 + 6c_3x + 12c_4x^2 + 20c_5x^3 \quad (12)$$

With the assumption of the same bending moment, then

$$\begin{pmatrix} b_0 + b_1x + b_2x^2 \\ +b_3x^3 + b_4x^4 \end{pmatrix} \begin{pmatrix} 2s_2 + 6s_3x \\ +12s_4x^2 + 20s_5x^3 \end{pmatrix} = \begin{pmatrix} b_0 + b_{1n}x + b_{2n}x^2 \\ +b_{3n}x^3 + b_{4n}x^4 \end{pmatrix} \begin{pmatrix} 2s_2 + 6c_3x \\ +12c_4x^2 + 20c_5x^3 \end{pmatrix} \quad (13)$$

The new coefficients of the width distribution can be obtained from the old ones combined with the data from the FEM simulation. (Skip the high order terms)

$$\begin{aligned} b_{1n} &= b_1 + 3b_0(s_3 - c_3)/s_2 \\ b_{2n} &= b_2 + 3(b_1s_3 - b_{1n}c_3)/s_2 + 6b_0(s_4 - c_4)/s_2 \\ b_{3n} &= b_3 + 3(b_2s_3 - b_{2n}c_3)/s_2 + 6(b_1s_4 - b_{1n}c_4)/s_2 \\ &\quad + 10b_0(s_5 - c_5)/s_2 \\ b_{4n} &= b_4 + 3(b_3s_3 - b_{3n}c_3)/s_2 + 6(b_2s_4 - b_{2n}c_4)/s_2 \\ &\quad + 10(b_1s_5 - b_{1n}c_5)/s_2 \end{aligned} \quad (14)$$

### OPTIMIZED WIDTH PROFILE EQUATIONS WITH FEM ANALYSIS ITERATION

At first, a bending HFM model was built with the width coefficients derived from the single mirror plate Eq. (6) without bending arms and applied for FEM simulation as in Fig. 4. The initial polynomial equation applied to the drawing as in Fig. 2 shows a focus size is 5.7mm.

However, the HFM focus size down to 0.32mm with 1 iteration, and after 4 iterations, the focus size down to 0.0138mm and the optimized width distribution polynomial coefficients were obtained from Eq. (14) as shown in Table 2.

With the same procedures, the VFM focus size down to 0.0214mm after 3 iterations and the optimized width distribution polynomial coefficients were also listed in Table 2. The thickness of both mirror is 7mm.

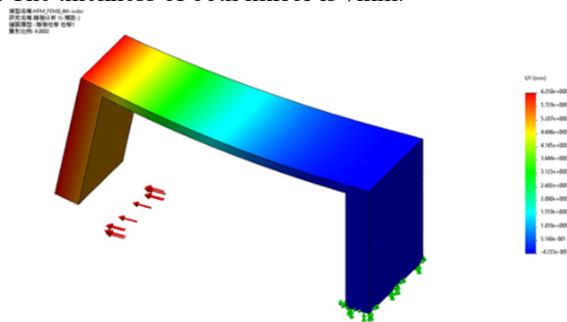


Figure 4: Initial HFM model for FEM simulation.

Table 2: The Coefficients of Width Profile Polynomial

Coef.	Bendable HFM	Bendable VFM
$b_0$	80	90
$b_1$	0.061754	-0.00753143
$b_2$	-6.527695E-04	-5.873345E-04
$b_3$	8.294557E-7	-1.679582E-6
$b_4$	4.173849E-9	1.080632E-10

From the Shadow simulation, the focusing point distribution plot of the bending mirrors is quite the same as the ideal profile. The size in horizontal direction is about only 1/5 of the vertical direction shows more better concentration. Compared to TLS IR, the situation is also the same as shown in Fig. 5.

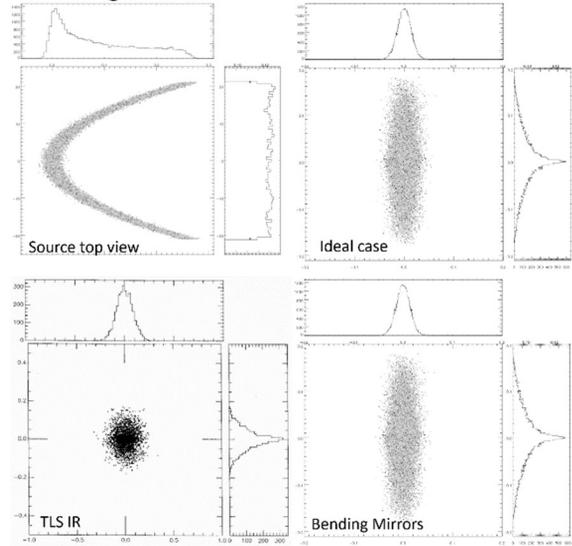


Figure 5: Shadow simulation of the bendable mirror.

### ENGINEERING DESIGN

With the width profile equations derived. The engineering design can be carried out. One pair of focusing mirror prototype design was finished as in Fig. 6 and waiting for the budget to be fabricated.

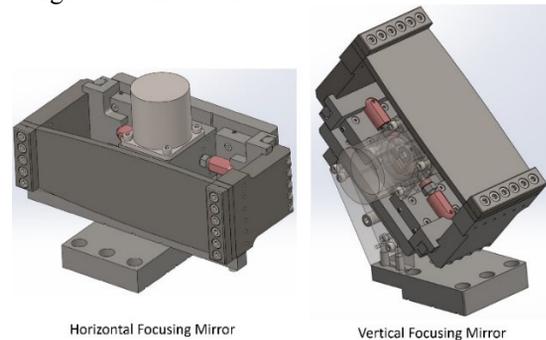


Figure 6: Prototype design of the two focusing mirror.

### CONCLUSION

By using the drawing method, a fine way to define a correction polynomial equation for focusing arc source is identified. With the pseudo moment and FEM simulation iteration procedures, the width profile equation of a plane mirror can be found to define a desired mirror surface shape with pure bending moment (one single force). The shadow simulation result shows good condition.

With Exact beam size correction in the vertical direction, The VFM focusing might be further improved if demanded.

The Engineering design is on the way and waiting for the budget to be fabricated since the TPS IR beamline is still not into schedule.

## REFERENCES

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